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Қ. И. Сәтпаев атындағы Қазақ ұлттық техникалық зерттеу университеті

# Х А Б А Р Л А Р Ы

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## ИЗВЕСТИЯ

НАЦИОНАЛЬНОЙ АКАДЕМИИ НАУК  
РЕСПУБЛИКИ КАЗАХСТАН  
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## NEWS

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*Қазақстан Республикасы Ұлттық ғылым академиясы "ҚР ҰҒА Хабарлары. Геология және техникалық ғылымдар сериясы" ғылыми журналының Web of Science-тің жаңаланған нұсқасы Emerging Sources Citation Index-те индекстелуге қабылданғанын хабарлайды. Бұл индекстелу барысында Clarivate Analytics компаниясы журналды одан әрі the Science Citation Index Expanded, the Social Sciences Citation Index және the Arts & Humanities Citation Index-ке қабылдау мәселесін қарастыруда. Web of Science зерттеушілер, авторлар, баспашылар мен мекемелерге контент тереңдігі мен сапасын ұсынады. ҚР ҰҒА Хабарлары. Геология және техникалық ғылымдар сериясы Emerging Sources Citation Index-ке енуі біздің қоғамдастық үшін ең өзекті және беделді геология және техникалық ғылымдар бойынша контентке адалдығымызды білдіреді.*

*НАН РК сообщает, что научный журнал «Известия НАН РК. Серия геологии и технических наук» был принят для индексирования в Emerging Sources Citation Index, обновленной версии Web of Science. Содержание в этом индексировании находится в стадии рассмотрения компанией Clarivate Analytics для дальнейшего принятия журнала в the Science Citation Index Expanded, the Social Sciences Citation Index и the Arts & Humanities Citation Index. Web of Science предлагает качество и глубину контента для исследователей, авторов, издателей и учреждений. Включение Известия НАН РК. Серия геологии и технических наук в Emerging Sources Citation Index демонстрирует нашу приверженность к наиболее актуальному и влиятельному контенту по геологии и техническим наукам для нашего сообщества.*

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margulan.ibraimov@kaznu.kz, magzomxzn@gmail.com**THE DEVICE FOR MULTIPLYING POLYNOMIALS MODULO  
AN IRREDUCIBLE POLYNOMIAL**

**Abstract.** In this paper a design of the polynomial multiplier by modulo of irreducible polynomial with coefficients in GF(2) is described. The main advantages of using the non-positional notations and the main directions of the development of the modular number systems are described. The work is implemented in the framework of a research on the hardware implementation of encryption algorithm based on polynomial residue number system.

**Keywords:** hardware encryption, binary polynomials, residue number system, hardware multiplier.

**Introduction.** As a result of the search for ways of increasing the efficiency of electronic computing devices, methods for detecting and correcting errors and creating highly reliable computer systems, research began in the field of nonpositional notation systems in the middle of the 20th century.

In the classical positional number system, the value of each digit in the number designation depends on its position. In contrast, in nonpositional numeration systems, the designation of numbers is based on other principles.

An example of a nonpositional system widely used in computing technologies is the residual number system (RNS) [1]. In RNS an multi-valued integer in the positional notation is represented as a sequence of several positional numbers of a small digit.

The first thought about the possibility of using RNS in computing technology was expressed by Valach and Svoboda [2]. In 1968 Akushsky and Yuditsky published the book "Machine arithmetic in residual classes" [3], a fundamental work on new machine arithmetic. They took an active part in the implementation of a specialized electronic computer based on the RNS. In parallel, Garner published a paper that describes the system of residual classes and arithmetic operations in it [4].

From the mid-1970s, theoretical developments began to be applied in technological developments. More than 150 papers were published in the period from the mid-1970s to the mid-1980s in this direction, in the same period the first patents and books on RNS were obtained. Initially, the main scope of RNS was digital signal processing. Juan built and tested a matched filter in a two-dimensional RNS capable of operating 20 million operations per second [5].

In [6] an implementation of RNS with arrays of lookup tables placed with high density in read-only memory is considered. The implementation of such a system is limited to the operations of addition, subtraction, multiplication and scaling by a predefined constant. Particular attention is paid to the scaling algorithm, and the developed two scaling algorithms are described.

Currently, RNS is often used in the development of efficient and high-performance special-purpose processors [7]. For instance, some applications of nonpositional number systems considered for neural networks processing [8], which may have a high potential in our country for the implementation of high-

performance and protected artificial intelligence based systems [9], as well as for highly-parallel multi-agent computing systems [10].

RNS is widely used in cryptography. For example, modular arithmetic allows creating an effective hardware implementation of cryptographic systems [11]. The use of a nonpositional number system allows us to speed up slow computations in asymmetric encryption algorithms and increase their reliability. For example, RNS is widely used in the hardware implementation of RSA and ECC algorithms [12, 13].

In polynomial RNS in  $GF(2)$ , addition and subtraction operations are performed via bitwise XOR, so that no overflow problem occurs. In other words, a modular reduction is not necessary for operations of addition and subtraction. However, coercion to the module is still necessary for multiplication.

In [14-17] described approaches to the development of a block symmetric encryption algorithm based on polynomial RNS, where secrecy is determined by the so-called "full key", which consists of a secret key (pseudo-random) sequence and the selected moduli system. Resistance against exhaustive search in this case depends not only on the length of the secret sequence but also on the composition of selected system of polynomial bases, and on the number of possible permutations of bases in that system [18]. That is, the reliability of the encryption algorithm is increased by introducing additional secret parameters in the form of a base system of a polynomial RNS. This allows creating encryption systems with customized cryptographic strength, balanced between computing performance and security for different use cases.

In [19-22], various architectures and variants of the implementation of multipliers in the RNS are presented from the point of view of application in public-key cryptographic systems.

This article discusses the multiplier version of polynomials modulo irreducible polynomial for the previously described symmetric cryptosystems based on polynomial RNS, with the possibility of using base systems of arbitrary length and composition.

**The developed device of modular polynomials multiplier.** Consider the design of a device for multiplying the polynomial  $A(x)$  by the polynomial  $B(x)$  modulo an irreducible polynomial  $P(x)$  with calculating the remainder  $R$  by the formula:

$$R = [A(x) * B(x)] \text{mod } P(x),$$

where  $A(x)$  is a polynomial-multiplicand (further multiplicand),  $B(x)$  is a polynomial-multiplier (further multiplier), where  $P(x)$  is a polynomial-module (further module), where  $A(x) < P(x)$  and  $B(x) < P(x)$ .

The multiplication of polynomials modulo is performed in stages. The number of the binary representation of the multiplier -  $N$ , determines the number of stages of multiplication. At each stage, a partial remainder  $R_i$  is formed. The remainder calculated at the last stage  $R_{N-1}$  is the result of multiplying polynomials modulo. The formation of each partial remainder  $R_i$  in turn is performed in three sub-steps. In the first sub-step, the multiplicand  $A(x)$  is logically multiplied by the binary coefficient  $b_i$  of the multiplier  $B(x)$  starting from the highest bit. In the second sub-step,  $A(x)*b_i$  is summed modulo 2 with the previous intermediate partial remainder  $R_{i-1}$  shifted by one bit to the higher order and the value  $C_i = A(x) * b_i \oplus 2R_{i-1}$  is calculated, in the third sub-step partial remainder  $R_i$  by reducing  $C_i$  modulo  $P(x)$ .

Figure 1 shows a block diagram of a device for multiplying polynomials modulo an irreducible polynomial. The device consists of four blocks and one delay line 5. The register block - 1 consists of register  $RgP(x)$  for storing the module  $P(x)$ , register  $RgA(x)$  for storing the multiplicand  $A(x)$ , register  $RgB(x)$  for storing the multiplier  $B(x)$ . The block 2 consists of  $N$  AND gates ( $And_0 \div And_{N-1}$ ). The block 3 consists of adders modulo 2 ( $Add2_1 \div Add2_{N-1}$ ). The block 4 consists of partial remainder formers - 4 ( $PRF_1 \div PRF_{N-1}$ ).

At input 6, a polynomial is accepted - the module  $P(x)$ . Input 7 is designed to receive a polynomial - multiplicand  $A(x)$ . Input 8 is used to receive a polynomial - multiplier  $B(x)$ . The signal "Start" is fed through the input 9. 10 - is the output of the device.

The information outputs of register  $RgP(x)$  are connected by the first information inputs of all ( $PRF_1 \div PRF_{N-1}$ ). The information outputs of register  $RgA(x)$  are connected to the information inputs of the ( $And_0 \div And_{N-1}$ ) circuits, and the control inputs of the ( $And_0 \div And_{N-1}$ ) circuits are fed from the outputs of the  $RgB(x)$  register, respectively, the values of the bits  $b_{N-1}, b_{N-2}, \dots, b_1, b_0$  of the multiplier  $B(x)$ . The information outputs of the circuits  $And_0$  and  $And_1$  are supplied respectively to the first and second information inputs of the first modulo 2 adder  $Add2_1$ . The informational outputs of the  $Add2_1$  are connected to the second inputs  $PRF_1$ .

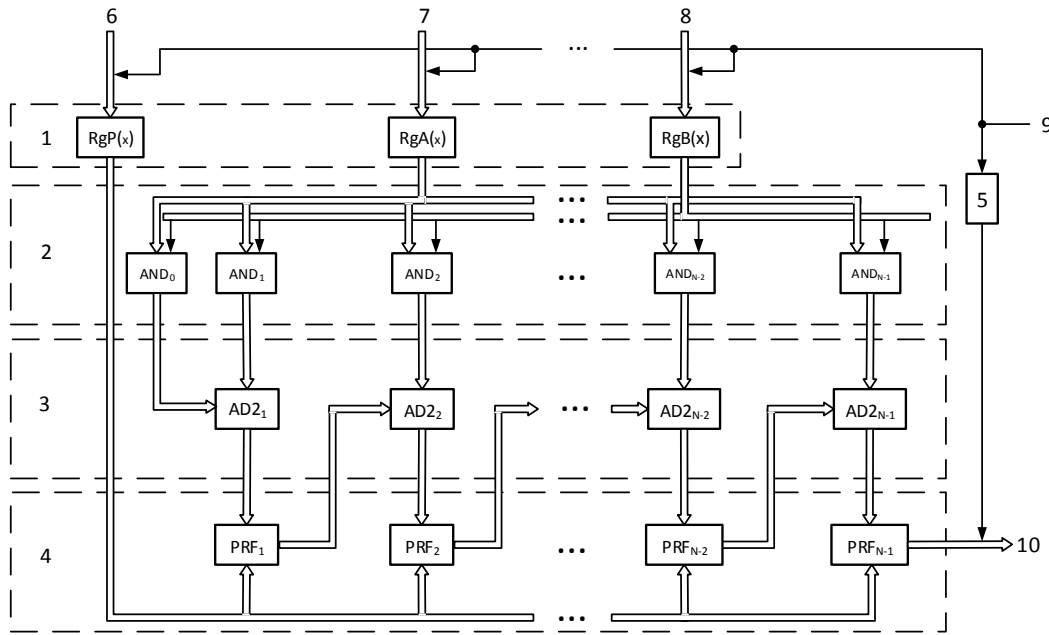


Figure 1 – Block diagram of the developed multiplier of polynomials by irreducible polynomial

The information outputs of the  $And_2, And_3, \dots, And_{N-2}, And_{N-1}$  circuits are connected to the second information inputs of  $Add_2, Add_3, \dots, Add_{N-2}, Add_{N-1}$ . To the first inputs of which are fed from the information outputs  $PRF_1, PRF_2, \dots, PRF_{N-2}, PRF_{N-1}$ , respectively. Information outputs  $Add_2, Add_3, \dots, Add_{N-2}, Add_{N-1}$  are connected to the second information inputs  $PRF_1, PRF_2, \dots, PRF_{N-2}, PRF_{N-1}$ , respectively. Information outputs  $PRF_1$  is connected to the output of device 10. A device for multiplying polynomials modulo an irreducible polynomials works as follows.

After applying the "Start" signal to input 9, binary representation of the polynomials  $P(x), A(x)$  and  $B(x)$  from inputs 6, 7, 8, are respectively accepted into registers  $RgP(x), RgA(x)$  and  $RgB(x)$ . From the outputs of register  $RgP(x)$ , binary representation of the module  $P(x)$  are fed to the first inputs  $PRF_1 \div PRF_{N-1}$ . Binary representation of the multiplicand polynomial  $A(x)$  are fed to the information inputs of the  $And_1 \div And_{N-1}$  circuits. From the information outputs of register  $RgB(x)$ , bits  $b_{N-1}, b_{N-2}, \dots, b_1, b_0$  of the binary representation of the multiplier are fed to the control inputs of the  $And_0 \div And_{N-1}$  circuits. If  $b_{N-1} = 1$ , then the code  $A(x)$  is generated at the output of the  $And_0$  circuit. Since  $A(x) < P(x)$ , the output  $R_0 = A(x)$  is formed at the output of  $And_0$ , which is fed to the second inputs of the adder modulo two  $Add_1$ , to the first input the result of the multiplication  $A(x) * b_{N-2}$  comes from the outputs  $And_1$ . At the output  $Add_1$ , we obtain  $C_1 = R_0 \oplus A(x) * b_{N-2}$ , which in turn is fed to the second input  $PRF_1$ . At the output, the first partial remainder  $R_1 = C \text{ mod } P(x)$  is formed, which is shifted by one bit towards the higher bit ( $2 * R_1$ ) is fed to the first input of the adder  $Add_2$  to the second input, which is fed from the output  $And_2$  the result is logically multiplied by  $A(x) * b_{N-3}$ . The output  $Add_2$  forms the values  $C_2 = A(x) * b_{N-3} * 2R_1$ , which is fed to the second inputs  $PRF$ , where  $R_2 = C \text{ mod } P(x)$  is formed. Similarly,  $R_3, R_4, \dots, R_{N-1}$  and  $C_3, C_4, \dots, C_{N-1}$  are formed. The delayed signal "Start" on delay lines - 5 outputs the result to the output 10 of the device. The magnitude of the delay on delay line 5 determined by the time of formation of the result.

Figure 2 shows a fragment of the functional diagram of the device, where partial remainders  $R_0$  и  $R_1$  are formed.

The binary representations of the polynomial  $A(x)$  and the bit  $b_{N-1}$  and  $b_{N-2}$  of the multiplier  $B(x)$  are fed to the information inputs of the  $And_0, And_1$  gates. When  $b_{N-1} = b_{N-2} = 1$ ,  $A(x) = R_0$  is formed at the output of the  $And_0$  and we get  $A(x)$  at the output of the  $And_1$ .  $A(x)$  is fed to the inputs of the adder modulo 2  $Add_1$  and calculates the value  $C_1$ , which is fed to the inputs  $Add_1$  and inputs of the circuit  $And_2$ , which are part of  $PRF_1$ . The highest bit  $C_1$  is fed to the control input of the circuit  $And_1$  and through the inverter NOT to the control input of the circuit  $And_2$   $PRF_1$ . When  $C_{highest} = 1$ , the result of summing



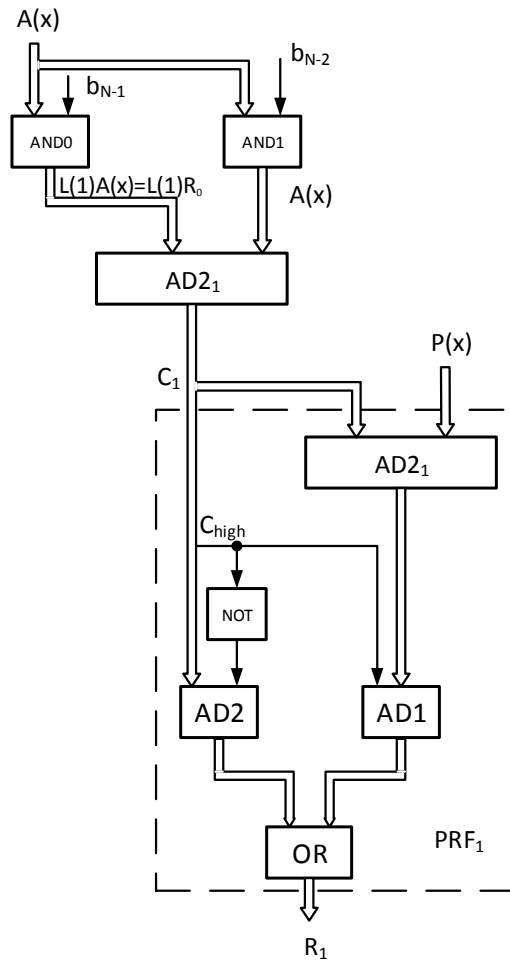


Figure 2 – Functional diagram of the partial residue Ri former

$C_1$  with  $P(x)$  modulo 2 from the output  $Add2_1$  through the circuits  $And_1$  and OR is transmitted to the output  $PRF_1$  forming  $R_1$  (with  $C_1 > P(x)$ ).

Consider the operation of the multiplication device of polynomials  $A(x)$  and  $B(x)$  modulo irreducible polynomials  $P(x)$  with a specific example.

Let  $A(x)=x^7 + x^5 + x^2 + x$ , the binary representation:  $A(x) = 10100110$ ;  $B(x)=x^7 + x^6 + x^5 + x + 1$ , binary representation:  $B(x) = 11100011$ ;  $P(x) = P(x)=x^8 + x^4 + x^3 + x + 1$ , binary representation:  $P(x) = 100011011$ ;

Since  $A(x) < P(x)$ , the zero partial product (remainder) -  $r_0=10100110$ .

The procedure for calculating residues is given in table 1.

Check example shown in figure 3.

$$\begin{array}{r}
 (x^4 + x + 1)(x^4 + x^2 + 1) = x^8 + x^6 + x^5 + x^3 + x^2 + x + 1. \\
 \oplus \begin{array}{r}
 x^8 + x^6 + x^5 + x^3 + x^2 + x + 1 \\
 x^8 + \quad x^5 + x^3 \\
 \hline
 x^6 + x^2 + x + 1 \\
 \oplus x^6 + x^3 + x \\
 \hline
 R(x) = x^3 + x^2 + 1,
 \end{array}
 \end{array}$$

Figure 3 – Example of the modular multiplication

The result corresponds to the binary representation - 01101.

Stages of hardware multiplication on the developed design

Stages	$b_i$	$2R_{i-1}, A(x) * b_i$	CM2	ФЧО
1	$b_0 = 1$ $b_1 = 0$	$R_0 = A(x) * b_0 = 10011$ $2R_0 = 100110$ $A(x) * b_1 = 0$	$C_1 = 2R_0 \oplus A(x) * b_1 = 2R_0 \oplus 0 = 100110$	$R_1 = C_1 \text{mod} P(x)$ $C_1 = 100110$ $P(x) = \oplus \underline{100101}$ $R_1 = 00011$
2	$b_2 = 1$	$2R_1 = 000110$ $A(x) * b_2 = 10011$	$C_2 = 2R_1 \oplus A(x) * b_2$ $2R_1 = 000110$ $A(x) = \oplus \underline{10011}$ $C_2 = 010101$	$R_2 = C_2 \text{mod} P(x)$ $C_2 = \oplus 010101$ $P(x) = \oplus \underline{100101}$ $R_2 = 010101$
3	$b_3 = 0$	$2R_2 = 101010$ $A(x) * b_3 = 0$	$C_3 = 2R_2 \oplus 0$ $C_3 = 101010$	$R_3 = C_3 \text{mod} P(x)$ $C_3 = 101010$ $P(x) = \oplus \underline{100101}$ $R_3 = 001111$
4	$b_4 = 1$	$2R_3 = 011110$ $A(x) * b_4 = 10011$	$C_4 = 2R_3 \oplus A(x)$ $2R_3 = 011110$ $A(x) = \oplus \underline{10011}$ $C_4 = 001101$	$R_4 = C_4 \text{mod} P(x)$ $C_4 = 001101$ $P(x) = \oplus \underline{100101}$ $R_4 = 01101$

**Results.** From the considered device of modular multiplication of polynomials it is not difficult to notice that at each stage of multiplication for calculating the partial remainder the polynomials are processed on two modulo-two adders in which there are no carry bit transfer, which allows constructing high-speed modulus multipliers.

In the proposed multiplication scheme, if the obtained remainder  $R(x)$  is multiplied by the inverse value of  $B^{-1}(x)$ , then the value of the polynomial  $A(x)$  can be restored. This makes it possible to implement on such multipliers high-speed devices for encrypting and decrypting data.

**Conclusion.** As was shown above, the main advantages of using the nonpositional number system are the absence of transfer of bits in the operations of addition and multiplication, and, consequently, the possibility of parallel execution of operations on each of the bases of the system, which significantly speeds up the calculation process. For the most effective implementation of computing devices based on the residual class system, it is required to develop non-standard circuit solutions that effectively perform calculations in the nonpositional number system.

The developed modular multiplier is to be used as a main computation unit in hardware implementation of the considered symmetric encryption system built on polynomial RNS.

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#### КЕЛТІРІЛМЕЙТІН КӨПМҮШЕЛІКТЕР МОДУЛІ БОЙЫНША КӨПМҮШЕЛІКТЕРДІ КӨБЕЙТУ ҚҰРЫЛҒЫСЫ

**Аннотация.** Мақалада коэффициенттері  $GF(2)$  келтірілмейтін көпмүшеліктер модулі бойынша көпмүшеліктерді көбейту құрылғысының құрылысы баяндалады. Бейпозициялық санақ жүйесін қолданудың негізгі артықшылықтары және модульдік сандар жүйесін дамытудың негізгі бағыттары сипатталған. Жұмыс поли-

номиальды қалдық класстар сандық жүйесі негізінде құрылған шифрлау алгоритмін аппараттық түрде іске асыру бойынша зерттеулер шеңберінде жүзеге асырылады.

**Түйін сөздер:** аппараттық шифрлау, екілік көпмүшеліктер, қалдық класстар жүйесі, аппараттық көбейткіш.

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### **УСТРОЙСТВО УМНОЖЕНИЯ ПОЛИНОМОВ ПО МОДУЛЮ НЕПРИВОДИМЫХ ПОЛИНОМОВ**

**Аннотация.** В статье описывается устройство умножения полиномов по модулю неприводимых полиномов с коэффициентами над  $GF(2)$ . Описаны основные преимущества использования непозиционных систем счисления и основные направления развития модульных систем счисления. Работа выполняется в рамках исследований по аппаратной реализации алгоритма шифрования, построенного на базе полиномиальной системы остаточных классов.

**Ключевые слова:** аппаратное шифрование, бинарные полиномы, система остаточных классов, программный умножитель.

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