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ГЕОЛОГИЯ ЖӘНЕ ТЕХНИКАЛЫҚ ҒЫЛЫМДАР СЕРИЯСЫ

♦

СЕРИЯ ГЕОЛОГИИ И ТЕХНИЧЕСКИХ НАУК

♦

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Қазақстан Республикасы Ұлттық ғылым академиясы "ҚР ҰҒА Хабарлары. Геология және техникалық ғылымдар сериясы" ғылыми журналының Web of Science-тің жаңаланған нұсқасы Emerging Sources Citation Index-те индекстелуге қабылданғанын хабарлайды. Бұл индекстелу барысында Clarivate Analytics компаниясы журналды одан әрі the Science Citation Index Expanded, the Social Sciences Citation Index және the Arts & Humanities Citation Index-ке қабылдау мәселесін қарастыруда. Webof Science зерттеушілер, авторлар, баспашылар мен мекемелерге контент тереңдігі мен сапасын ұсынады. ҚР ҰҒА Хабарлары. Геология және техникалық ғылымдар сериясы Emerging Sources Citation Index-ке енуі біздің қоғамдастық үшін ең өзекті және беделді геология және техникалық ғылымдар бойынша контентке адалдығымызды білдіреді.

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COMPUTER MODELING OF CREEP FOR HEREDITARY MATERIALS BY ABEL'S KERNEL

Abstract. The work is devoted to the computer modeling of creep procedure for hereditary materials. Creep procedure is described by nonlinear integral equation of Yu. N. Rabotnov, and creep kernel is represented by Abel's kernel. New efficient method has been proposed for determining of parameters (α, δ) for Abel's kernel. Bisection method is used for obtaining of parameter α. Algorithm and relevant software have been developed for calculating of parameters α и δ.

Efficient methods and relevant software have been developed for calculation of values of instantaneous strain and creep strain for hereditary materials. High accuracy of modeling of creep procedure has been shown by means of developed methods and software based on materials Nylon 6 and glass-reinforced plastic TC 8/3-250.

Key words: creep, Abel's kernel, bisection method, instantaneous strain, creep strain.

1. INTRODUCTION. It is known that many natural and artificial materials under load action show their viscoelastic properties. Currently there are sufficiently developed experimental methods for evaluation of viscoelastic characteristics of materials, as well as theoretical methods for their description [1-3].

One of simple, but efficient means for description of hereditary materials deformation was proposed in 1948 by Yu.N. Rabotnov [4]. It was based on similarity of isochronous creep curves of materials. Meanwhile the process of creep strain is described by the equation of Boltzmann-Volterra for linear viscoelastic materials [5], but strain in the left part of the equation has been replaced by the so called "curve of instantaneous deformation", which is determined experimentally.

It is known that the selection of kernel of the integral equation and determination of its parameters is one of the most responsible actions in description of mechanical behavior of viscoelastic materials. In the works [6-11] Rabotnov's fractional exponential kernel and Abel's kernel have been used for description of creep procedure of various materials.

In Kazakhstan complex experimental and theoretical research for deformation and strength of asphalt concretes is carried out [9-20]. The current paper is continuation of the above works and it considers the matters of computer modeling for creep procedure of hereditary materials using nonlinear integral equation of Yu. N. Rabotnov with Abel's kernel.

2. NONLINEAR CREEP.

2.1. Nonlinear equation of creep. Considering similarity property of isochronous creep curves, Yu. N. Rabotnov proposed the following nonlinear integral equation to describe the process of nonlinear deformation of hereditary materials [4, 21-23]:

$$\varphi\left[\varepsilon(t)\right] = \sigma(t) + \int_0^t K(t-\tau)\,\sigma(\tau)\,d\tau, \tag{1}$$

where $\varepsilon(t)$ – is strain at time moment t; $\sigma(t)$ – is stress at time moment t; $\sigma(\tau)$ – is stress at time moment τ; $K(t-\tau)$ – is creep kernel; t – is observation time; τ – time preceding observation time t.

Expression $\varphi[\varepsilon(t)]$ in the left part of integral equation (1) represents by itself the so-called "instantaneous deformation curve".

2.2. Rabotnov's kernel. Creep kernel of integral equation (1) is described by Rabotnov's fractional exponential function [21-23]:

$$K(t-\tau)=\lambda\Im_{-\alpha}(-\beta,t-\tau)=\lambda(t-\tau)^{-\alpha}\sum_{n=0}^{\infty}\frac{(-\beta)^n\ (t-\tau)^{(1-\alpha)\mathrm{n}}}{\Gamma[(1-\alpha)(1+n)]}, \tag{2}$$

where $\Im_{-\alpha}(-\beta,t-\tau)$ – is Rabotnov's fractional exponential function; λ, α, β are creep kernel parameters (λ>0, 0< α<1, β>0); Γ(·) is gamma-function.

Inserting expression for creep kernel (2) into integral equation (1), and considering σ = const at creep, we obtain:

$$\varphi[\varepsilon(t)]=\sigma\left[1+\lambda\sum_{n=0}^{\infty}\frac{(-\beta)^n t^{(1-\alpha)(1+n)}}{\Gamma[(1-\alpha)(1+n)+1]}\right]. \tag{3}$$

2.3. Abel's kernel. Practically always one can accept that a relationship between stress and strain in materials is a linear one at small stresses. Therefore for determination of creep parameters one can use creep curve of a material at small stress and apply linear viscoelasticity approach [6].

As it is known creep curves of materials depending on stress level and temperature can have two or three characteristic strain sites [7-9]: site I with unstabilized creep, site II with stabilized creep and site III of accelerating creep.

Then the method is proposed, according to which for determination of creep parameters of a material α, ε_0 and δ it will be sufficient to consider only site I of creep curve.

Having accepted n=0, from equation (3) we will obtain:

$$\varphi[\varepsilon(t)]=\sigma\left[1+\frac{\delta\ t^{(1-\alpha)}}{(1-\alpha)}\right]. \tag{4}$$

One can see that the right part of the obtained equation contains a well-known Abel's function with unknown parameters α and δ.

Having divided both parts of the equation (4) into instantaneous elasticity modulus E_0 for the case of linear deformed material, we will obtain:

$$\varepsilon(t)=\varepsilon_0\left[1+\frac{\delta}{1-\alpha}t^{(1-\alpha)}\right], \tag{5}$$

where ε_0 is instantaneous strain.

2.4. Abel's kernel parameters. Equation (5) contains three unknown parameters ε_0, α and δ. As it has been said above, in the right part it contains known Abel's function with parameter of singularity α, which has the value within the interval (0, 1). In the works [7, 9-11] it was proposed to consider the parameter α as known and to determine the unknown parameters ε_0 and δ with the use of least square method. According to the least square method the values of parameters ε_0 and δ should meet the following condition:

$$S(\varepsilon_0,\delta)=\sum_{i=1}^{m}\left[\varepsilon_0\left(1+\frac{\delta}{1-\alpha}t_i^{(1-\alpha)}\right)-\varepsilon_{ei}\right]^2\to\min, \tag{6}$$

where $S(\varepsilon_0,\delta)$ is sum of squares of deviations; ε_{ei} are values of creep strain determined experimentally; m is number of creep strains.

From the following two partial derivatives $\frac{\partial S(\varepsilon_0,\delta)}{\partial\varepsilon_0}=0$ и $\frac{\partial S(\varepsilon_0,\delta)}{\partial\delta}=0$ one can find expressions for determining the parameters ε_0 and δ:

$$\varepsilon_0 = \frac{\sum_{i=1}^{m} \varepsilon_i \sum_{i=1}^{m} t_i^{2(1-\alpha)} - \sum_{i=1}^{m} t_i^{(1-\alpha)} \sum_{i=1}^{m} \varepsilon_i t_i^{(1-\alpha)}}{m \sum_{i=1}^{m} t_i^{2(1-\alpha)} - \left[\sum_{i=1}^{m} t_i^{(1-\alpha)} \right]^2}, \tag{7}$$

$$\delta = \frac{\sum_{i=1}^{m} \left(\frac{\varepsilon_i}{\varepsilon_0} - 1 \right) t_i^{(1-\alpha)}}{\frac{1}{1-\alpha} \sum_{i=1}^{m} t_i^{2(1-\alpha)}}. \tag{8}$$

Selecting sequentially the values for parameter α from interval (0, 1) with specific step, from equation (7) we can find the values for parameter $\varepsilon_0 = \varepsilon_0(\alpha)$. Inserting the obtained values for parameter ε_0 and corresponding values for parameter of singularity α into expression (8), we can determine the values for parameter $\delta = \delta(\varepsilon_0, \alpha)$.

Then sequentially inserting the obtained values for parameters α, ε_0 and δ into equation (5), we can calculate the values for creep strain $\varepsilon(t_i) = \varepsilon_i(t_i, \alpha, \varepsilon_0, \delta)$.

Calculating under the formula

$$\Delta\varepsilon_i = \frac{\varepsilon_i(t_i, \alpha, \varepsilon_0, \delta) - \varepsilon_{ei}(t_i)}{\varepsilon_{ei}(t_i)} \cdot 100\,\% \tag{9}$$

the deviations of the calculated values for creep strain from those obtained experimentally, one can select optimum values for parameters α, ε_0 and δ, providing the least value of $\Delta\varepsilon_i$.

3. A NEW METHOD FOR DETERMINING OF ABEL'S KERNEL PARAMETERS.

3.1. Modified equations. The analysis of numerous experimental and calculated results [11] has shown that model ε_0^m and experimental ε_0^e instantaneous strains practically always coincide with high accuracy. Therefore further we accept that

$$\varepsilon_0^m \approx \varepsilon_0^e \tag{10}$$

Considering equation (10) the expressions (7) and (8) can be represented in the form of:

$$1 - \frac{\sum_{i=1}^{m} k_e(t_i) \sum_{i=1}^{m} t_i^{2(1-\alpha)} - \sum_{i=1}^{m} t_i^{(1-\alpha)} \sum_{i=1}^{m} k_e(t_i) t_i^{(1-\alpha)}}{m \sum_{i=1}^{m} t_i^{2(1-\alpha)} - \left[\sum_{i=1}^{m} t_i^{(1-\alpha)} \right]^2} = 0, \tag{11}$$

$$\delta = \frac{\sum_{i=1}^{m} \left[k_e(t_i) - 1 \right] t_i^{(1-\alpha)}}{\frac{1}{1-\alpha} \sum_{i=1}^{m} t_i^{2(1-\alpha)}}. \tag{12}$$

In the expressions (11) and (12) $k_e(t)$ represents by itself the so called experimental rheological parameter [11], determined under formula:

$$k_e(t) = \frac{\varepsilon_e(t)}{\varepsilon_0^e}, \tag{13}$$

where $\varepsilon_e(t)$ is value of creep strain at time moment *t*, determined experimentally; ε_0^e is instantaneous strain at time moment *t=0*, determined experimentally.

Experimental rheological parameter $k_e(t)$ represents by itself normalized time function in relation to experimental instantaneous strain. It has the value equal to 1 at *t=0* and more than 1 at time values of *t*>0. It shows in how many times the experimental values of creep strain are more at different time values compared with instantaneous strain, which has been also obtained experimentally.

3.2. Bisection method. As it is seen, expression (11) contains only one unknown parameter α. From the mathematical point of view the unknown parameter α is root of equation (11) and to obtain α one can use one of approximate methods. In our case bisection method (method of dichotomy, half division) is used [24].

It is known in advance that parameter of creep kernel α has the value in the interval (0.1) [21-23]. The essence of the bisection method is in the fact that a segment where the root is being found is divided in half, and the half is taken on the ends of which the function has the value of opposite signs as before.

We will write the expression in the following form:

$$f(\alpha) = 1 - \frac{\sum_{i=1}^{m} k_e(t_i) \sum_{i=1}^{m} t_i^{2(1-\alpha)} - \sum_{i=1}^{m} t_i^{(1-\alpha)} \sum_{i=1}^{m} k_e(t_i) t_i^{(1-\alpha)}}{m\sum_{i=1}^{m} t_i^{2(1-\alpha)} - \left[\sum_{i=1}^{m} t_i^{(1-\alpha)}\right]^2}. \quad (14)$$

Then the procedure for calculating of value for parameter α can be performed under the following algorithm:

1. Inserting of values for time t_i, for the experimental values of creep strain ε_i and given accuracy $\bar{\varepsilon}$.
2. To choose segment [*a, b*] in the interval α ϵ (0.1).
3. Calculating of the center of the segment [*a, b*] under formula:

$$c = \frac{a+b}{2}. \quad (15)$$

4. Calculating of value for function *f(α)* at $\alpha = c$, i.e. *f(c)*.
5. Check-up of condition:

$$f(c) \le \bar{\varepsilon}. \quad (16)$$

6. If condition (15) is true, then the central value *c* of the segment [*a, b*] is accepted as the value of parameter *α*. If not, the segment [*a, b*] is divided into halves and for further calculations we take one from two halves, on the ends of which the function *f(α)* has values of opposite signs, and calculations are performed again from 3.

7. As soon as the value of parameter α is found, the value of parameter δ is calculated under expression (12).

Figure 1 represents block diagram of the software, performing the calculation procedure for values of creep kernel parameter α under the abovementioned algorithm.

4. CALCULATION OF MODEL CREEP CURVE.

4.1. Short calculation method. Using approach developed in the authors' work [11], one can determine model (theoretical, calculated) values for creep strain in the following sequence:

1. Calculation of values for model rheological parameter $k_m(t)$ under formula:

$$k_m(t) = 1 + \frac{\delta}{1-\alpha} \cdot t^{(1-\alpha)}. \quad (17)$$

2. Determination of instantaneous strain at stresses σ:

$$\bar{\varepsilon}_0^m(\sigma) = \frac{1}{m} \sum_{i=1}^{m} \frac{\varepsilon_e(t_i)}{k_m(t_i)}. \quad (18)$$

3. Calculation of model values for creep strain at stresses σ:

$$\varepsilon_m(t) = \bar{\varepsilon}_0^m(\sigma) \cdot k_m(t). \quad (19)$$

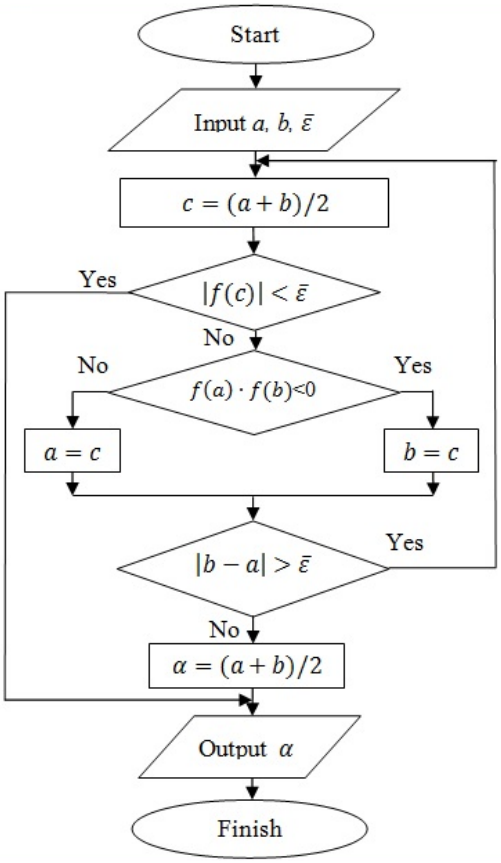

Figure 1 – Block diagram

4.2. Examples.

4.2.1. Material Nylon 6. The works [25, 26] contain test results for material Nylon 6 at stresses 5, 10 and 15 MPa. For all stresses the duration of experiment was 100 hours. Creep strain values for material Nylon 6 at the above stresses obtained by processing of experimental results are represented in table 1.

Table 1 – Creep strain values for material Nylon 6

Time t, h	Strain ε_e (t), %, at stress σ, MPa		
	5	10	15
0	0.1537	0.3873	0.6650
1	0.4200	1.0585	1.8174
20	0.5321	1.3408	2.3022
40	0.5621	1.4164	2.4319
60	0.5804	1.4624	2.5110
80	0.5937	1.4961	2.5689
100	0.6043	1.5229	2.6148

By calculating under the developed software the following values for parameter of creep kernel have been determined: α = 0.8860; δ = 0.1984.

Figure 2 shows the values of experimental rheological parameter and the graph of model rheological parameter. As it is seen, experimental and model rheological parameters coincide with high accuracy. Maximal deviation is equal to 0.26%.

Experimental and calculated values for creep strain of the material Nylon 6 at three stresses are shown in figure 3. It is clearly seen that the degree of coincidence for calculated strains with experimental ones is high.

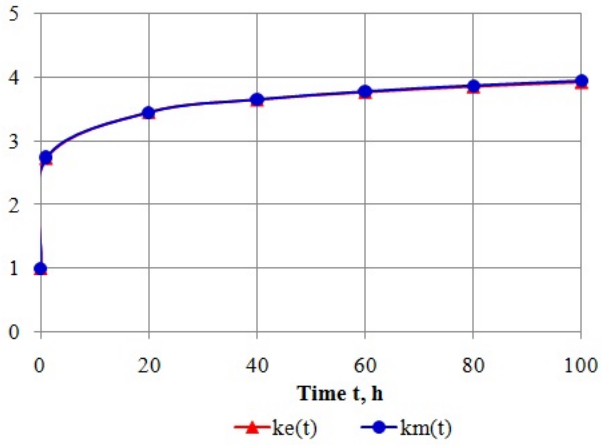

Figure 2 – Experimental and model rheological parameters of material Nylon 6

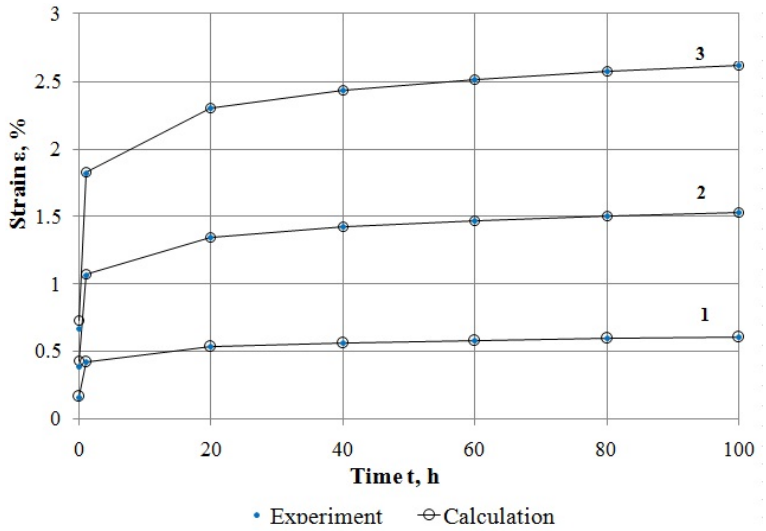

Figure 3 – Creep curves of material Nylon 6 at various stresses: 1-5 MPa; 2-10 MPA; 3-15 MPA

4.2.2. Glass-reinforced plastic TC 8/3-250. In the work [27] samples of glass-reinforced plastic TC 8/3-250 have been tested for creep at the temperature of 23.5 ± 2°C. As glass-reinforced plastic is an anisotropic material the samples have been cut perpendicularly to textile warp (Θ = 0°), along textile warp (Θ = 90°) and at an angle of (Θ = 45°) to textile warp.

Experimental values for creep strain of glass-reinforced plastic at various stresses are given in tables 2–4.

Table 2 –Creep strain values of glass-reinforced plastic (Θ = 0°)

Time t, h	Strain $\varepsilon_e(t)$, %, at stress σ, MPa					
	40.1	80.2	120.3	160.4	200.5	240.6
0	0.1822	0.3855	0.6140	0.8987	1.2111	1.5219
1	0.1948	0.4121	0.6564	0.9608	1.2948	1.6271
100	0.2051	0.4339	0.6911	1.0116	1.3632	1.7131
200	0.2068	0.4375	0.6968	1.0199	1.3745	1.7272
400	0.2085	0.4411	0.7026	1.0284	1.3859	1.7415
600	0.2095	0.4433	0.7060	1.0334	1.3926	1.7500
800	0.2102	0.4447	0.7083	1.0367	1.3971	1.7557
1000	0.2107	0.4458	0.7100	1.0392	1.4004	1.7598
1368	0.2115	0.4474	0.7127	1.0431	1.4057	1.7665

Table 3 – Creep strain values of glass-reinforced plastic (Θ = 45°)

Time t, h	Strain ε_e (t), %, t stress σ, MPa					
	20.3	40.6	60.9	81.2	101.5	121.8
0	0.1074	0.1946	0.5805	1.3624	2.4430	3.9262
1	0.1302	0.2359	0.7037	1.6515	2.9614	4.7593
50	0.1457	0.2639	0.7873	1.8478	3.3134	5.3251
200	0.1518	0.2750	0.8204	1.9255	3.4527	5.5489
400	0.1548	0.2806	0.8370	1.9643	3.5222	5.6608
600	0.1566	0.2838	0.8466	1.9869	3.5629	5.7260
800	0.1579	0.2861	0.8533	2.0027	3.5912	5.7715
1000	0.1588	0.2878	0.8586	2.0150	3.6132	5.8068
1320	0.1600	0.2900	0.8649	2.0300	3.6401	5.8500

Table 4 – Creep strain values of glass-reinforced plastic (Θ = 90°)

Time t, h	Strain ε_e (t), %, at stress σ, MPa			
	104.7	209.4	279.2	349.0
0	0.3478	0.6957	0.9276	1.1595
1	0.3616	0.7232	0.9643	1.2054
10	0.3668	0.7337	0.9782	1.2228
50	0.3709	0.7419	0.9892	1.2365
100	0.3728	0.7457	0.9942	1.2428
200	0.3746	0.7493	0.9991	1.2489
300	0.3757	0.7515	1.0020	1.2525
400	0.3765	0.7530	1.0040	1.2550
500	0.3771	0.7542	1.0056	1.2570

Parameters of creep kernel have the following values:
Θ = 0°: α = 0.8863; δ = 0.0082,
Θ = 45°: α = 0.8874; δ = 0.0250,
Θ = 90°: α = 0.8825; δ = 0.0048.

Values of experimental rheological parameter and graphs of model rheological parameter are represented in figures 4–6. And the figures 7–9 show experimental and calculated values of creep strain of glass-reinforced plastic.

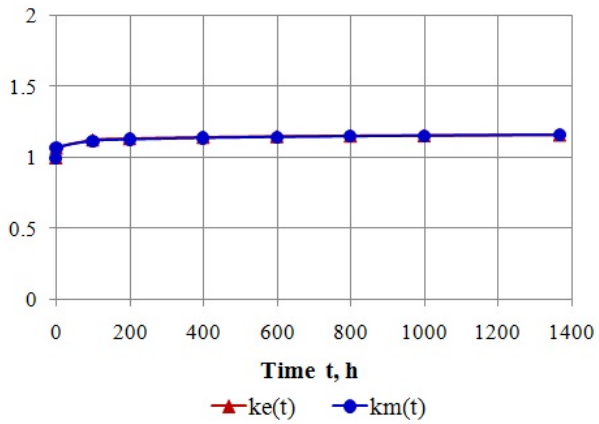

Figure 4 – Experimental and model rheological parameters of glass-reinforced plastic TC 8/3-250 (Θ = 0°)

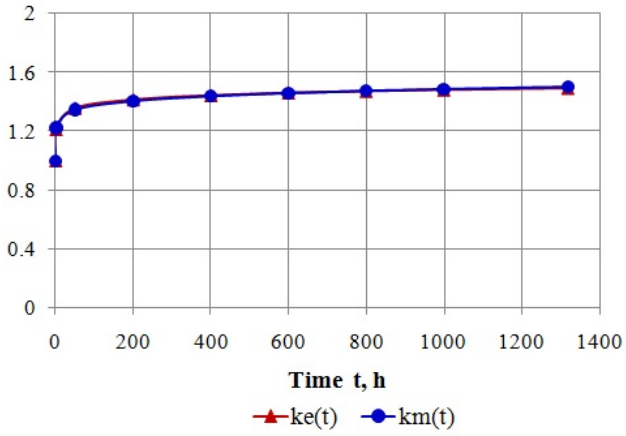

Figure 5 – Experimental and model rheological parameters of glass-reinforced plastic TC 8/3-250 (Θ = 45°)

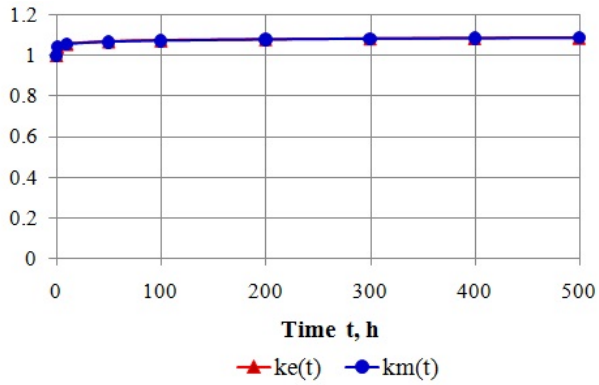

Figure 6 – Experimental and model rheological parameters of glass-reinforced plastic TC 8/3-250 (Θ = 90°)

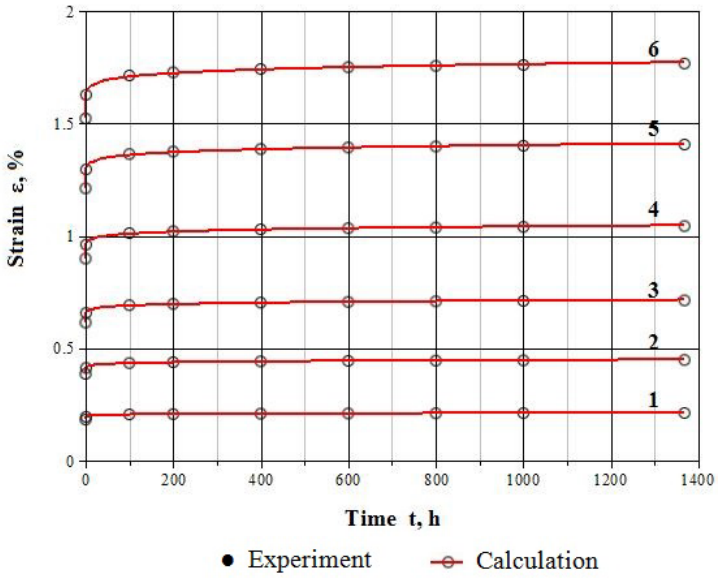

Figure 7 – Creep curves of glass-reinforced plastic TC 8/3-250 (Θ = 0°)
at various stresses: 1 – 40.1 MPA; 2 – 80.2 MPa; 3 – 120.3 MPa; 4 – 160.4 MPa; 5 – 200.5 MPa; 6 – 240.6 MPa

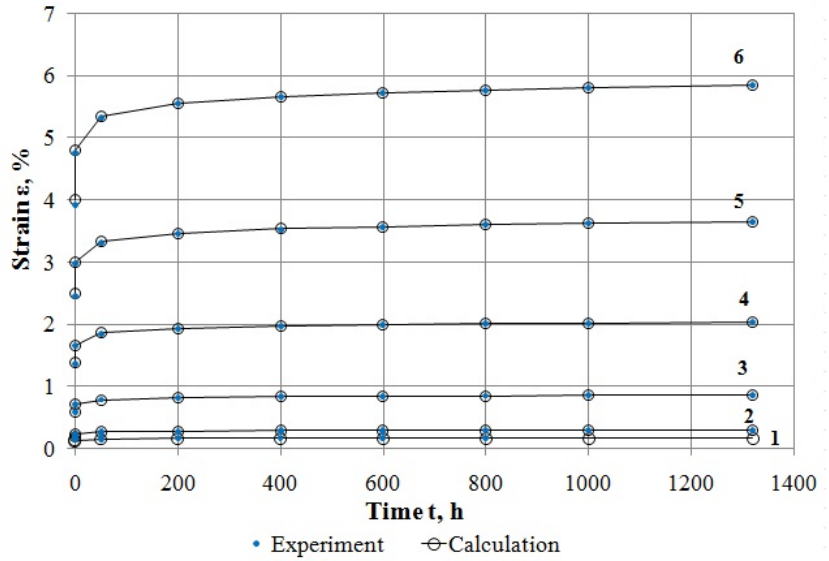

Figure 8 – Creep curves of glass-reinforced plastic TC TC 8/3-250 (Θ = 45°) at various stresses:
1 – 20.3 MPa; 2 – 40.6 MPa; 3 – 60.9 MPa; 4 – 81.2 MPa; 5 – 101.5 MPa; 6 – 121.8 MPa

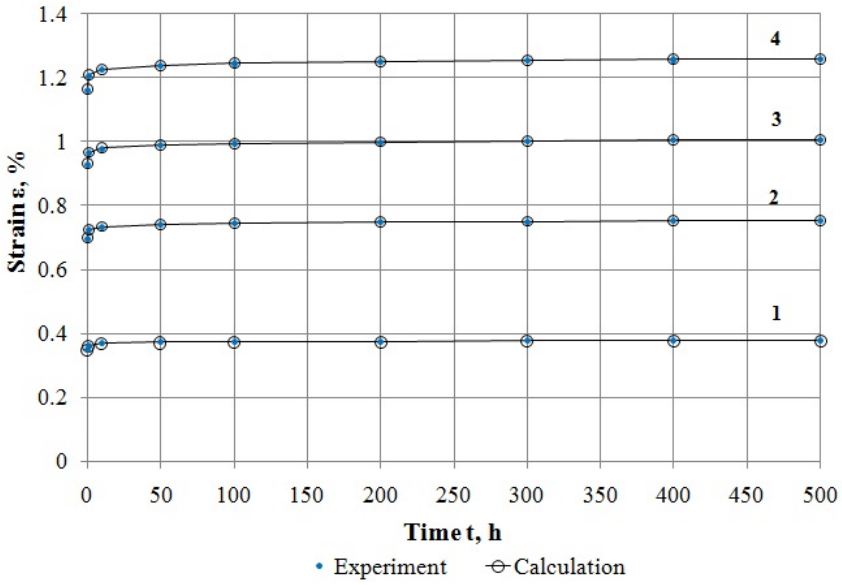

Figure 9 – Creep curves of glass-reinforced plastic TC 8/3-250 (Θ = 90°) at various stresses:
1 – 104.7 MPa; 2 – 209.4 MPa; 3 – 279.2 MPa; 4 – 349.0 MPa

Analysis of the constructed graphs shows that calculated curves coincide with relevant experimental data with high accuracy. Maximal deviation of model rheological parameter from an experimental one is 0.84%.

Thus, the abovementioned examples have shown that the developed methods and software allow determining of parameters for Abel's kernel, values of instantaneous strain and creep strain for hereditary materials with high accuracy.

Conclusion.

1. A new efficient method for determining of parameters (α, δ) for Abel's kernel has been proposed for description of hereditary materials creep by Yu. N. Rabotnov's nonlinear integral equation. Bisection method is used to obtain parameter α.

2. Algorithm and relevant software have been developed for calculating of values for parameters of creep α и δ with high accuracy.

3. An efficient method and relevant software have been developed for calculation of values for instantaneous strain and creep strain of hereditary materials.

4. High accuracy of creep procedure modeling has been shown on the examples of materials Nylon 6 and glass-reinforced plastic TC 8/3-250, which is provided by the developed method and relevant software.

REFERENCES

[1] Cristensen R.M. Theory of viscoelasticity: An introduction, Academic Press, New York, USA, 1971 (in Eng.).

[2] Tschoegl N.N. The phenomenological theory of linear viscoelastic behavior: An introduction, Springer-Verlag, Berlin, Germany, 1989 (in Eng.).

[3] Ferry J.D. Viscoelastic properties of polymers. Willey, New York, USA, 3rd edition, 1980 (in Eng.).

[4] Rabotnov Yu.N. Balance of the elastic medium with a aftereffect // Applied Mathematics and Mechanics. 1948. Vol. 12, issue 1. P. 53-62 (in Eng.).

[5] Volterra V. Lecons surles fonctions de lignes. Paris: Gautheir-Villard, 1913 (in Eng.).

[6] Iskakbayev A., Teltayev B., Alexandrov S. Determining the creep parameters of linear viscoelastic materials. Journal of Applied Mathematics. 2016. P. 1-6 (in Eng.).

[7] Iskakbayev, A., Teltayev, B., Andriadi, F., Estayev, K., Suppes, E., Iskakbayeva, A. Experimental research of creep, recovery and fracture processes of asphalt concrete under tension // Proceedings of the 24th International Congress on Theoretical and Applied Mechanics. Montreal, Canada, 21-26 August 2016 (in Eng.).

[8] Iskakbayev A.I., Teltayev B.B., Rossi C.O. Deformation and strength of asphalt concrete under static and step loadings // Proceedings of the AIIT International Congress on Transport Infrastructure and Systems (TIS 2017). Rome, Italy, 2017. P. 3-8 (in Eng.).

[9] Iskakbayev A., Teltayev B., Rossi C.O. Steady-state creep of asphalt concrete // Applied Sciences. 2017. 7. P. 1-13 (in Eng.).

[10] Teltayev B.B., Iskakbayev A., Oliviero Rossi C. Regularities of creep and long-term strength of hot asphalt concrete under tensile // Functional Pavement Design – Proceedings of the 4th Chinese-European Workshop on Functional Pavement Design. CEW. 2016. P. 169-178 (in Eng.).

[11] Iskakbayev A., Teltayev B., Rossi C.O., Yensebayeva G. Determination of nonlinear creep parameters for hereditary materials // Applied Sciences. 2018. P. 1-17 (in Eng.).

[12] Iskakbayev A., Teltayev B., Rossi C.O., Yensebayeva G. Experimental investigation of an asphalt concrete deformation under cyclic loading // News of the National Academy of Sciences of the Republic of Kazakhstan. Series of Geology and Technical Sciences. 2018. Vol. 2(428). P. 114-111 (https://doi.org/10.32014/2018.2518-170X), ISSN 2518-170X (Online), ISSN 2224-5278 (Print) (in Eng.).

[13] Iskakbayev A., Teltayev B., Rossi C.O. Modeling of cyclic strength for the asphalt concrete considering damage accumulation // Applied Sciences. 2017. 7(12) 1270 (in Eng.).

[14] Teltayev B.B. Fatigue destruction of asphalt concrete pavement. 2. Thermodynamics, 2017 // News of the National Academy of Sciences of the Republic of Kazakhstan. Series of Geology and Technical Sciences. 2018. Vol. 4(424). P. 148-169 (https://doi.org/10.32014/2018.2518-170X), ISSN 2518-170X (Online), ISSN 2224-5278 (Print) (in Eng.).

[15] Teltayev B.B. Fatigue failure of asphalt concrete pavement. 1. Self-organization and mechanical interpretation // News of the National Academy of Sciences of the Republic of Kazakhstan. Series of Geology and Technical Sciences. 2017. Vol. 3(423). P. 256-275 (https://doi.org/10.32014/2018.2518-170X) (in Eng.).

[16] Teltayev B.B. Evaluation of fatigue characteristics of hot mix asphalt with polymer additives // News of the National Academy of Sciences of the Republic of Kazakhstan. Series of Geology and Technical Sciences. 2017. Vol. 1(421). P. 141-148 (https://doi.org/10.32014/2018.2518-170X) (in Eng.).

[17] Teltayev B.B., Amirbayev Y.D. Experimental evaluation of strength for asphalt and polymer modified asphalt concretes at low temperatures // News of the National Academy of Sciences of the Republic of Kazakhstan. Series of Geology and Technical Sciences. 2017. Vol. 1(421). P. 167-176 (https://doi.org/10.32014/2018.2518-170X) (in Eng.).

[18] Teeltayev B.B. Fresh approach to low temperature cracking in asphalt concrete pavement // News of the National Academy of Sciences of the Republic of Kazakhstan. Series of Geology and Technical Sciences. 2016. Vol. 5(419). P. 161-178 (https://doi.org/10.32014/2018.2518-170X) (in Eng.).

[19] Teltayev B., Radovskiy B. Low temperature cracking problem for asphalt concrete pavements in Kazakhstan. RILEM Bookseries 13. P. 139-144 (in Eng.).

[20] Iskakbayev A., Teltayev B., Alexandrov S. Determination of the Creep Parameters of Linear Viscoelastic Materials // Journal of Applied Mathematics. 2016. P. 6 (in Eng.).

[21] Rabotnov Yu.N. Mechanics of Deformed Solid Body. M.: Nauka, 1988 (in Rus.).

[22] Rabotnov Yu.N. Creep of structure elements. M.: Nauka, 1966 (in Rus.).

[23] Rabotnov Yu.N. Elements of hereditary mechanics of solids. M.: Nauka, 1977 (in Rus.).

[24] Korn G.A., Korn T.M. Mathematical handbook for scientists and engineers. Definition, theorems, and formulas for reference and review. New York, McGraw-Hill, 1968 (in Eng.).

[25] Suvorova Yu.V., Mosin A.V. Determination of parameters of the Rabotnov's fractional exponential function with use of integral transform and modern software // Problems of mechanical engineering and automation. 2002. N 4. P. 54-56 (in Eng.).

[26] Suvorova Yu.V., About Yu.N. Rabotnov's nonlinear hereditary equation and its applications // News of the Russian Academy of Sciences. Mechanics of solids. 2004. N 1. P. 174-181 (in Eng.).

[27] Rabotnov Yu.N., Papernik L.H., Stepanychev E.I. Nonlinear creep of glass-reinforced plastic TC 8/3-250 // Mechanics of polymers. 1971. N 3. P. 391-397 (in Eng.).

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МҰРАЛЫ МАТЕРИАЛДАРДЫҢ ЖЫЛЖЫМАЛЫЛЫҒЫН АБЕЛЬ ӨЗЕГІМЕН КОМПЬЮТЕРЛІК МОДЕЛЬДЕУ

Аннотация. Жұмыс мұралы материалдардың жылжымалылық үдерісін компьютерлік модельдеуге арналған. Жылжымалылық үдерісі Ю. Н. Работновтың сызықты емес интегралдық теңдеуімен, жылжымалылық өзегі Абель өзегімен сипатталады. Абель өзегінің параметрлерін (α, δ) анықтаудың жаңа тиімді әдісі берілген. α параметрін анықтау үшін бисекция әдісі пайдаланылады. α және δ параметрлерін санаудың алгоритмі және сәйкес компьютерлік бағдарламасы жасалған.

Мұралы материалдардың шартты лездік деформациясы мен жылжымалылық деформациясын санауға арналған тиімді әдіс пен сәйкес компьютерлік бағдарлама жасалған. Nylon 6 және ТС 8/3-250 шыныпластик мысалдары арқылы жасалған әдістер және компьютерлік бағдарламалар көмегімен жылжымалылық үдерісін модельдеудің жоғары дәлдігі көрсетілген.

Түйін сөздер: жылжымалылық, Абель өзегі, бисекция әдісі, шартты лездік деформация, жылжымалылық деформациясы.

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КОМПЬЮТЕРНОЕ МОДЕЛИРОВАНИЕ ПОЛЗУЧЕСТИ НАСЛЕДСТВЕННЫХ МАТЕРИАЛОВ ЯДРОМ АБЕЛЯ

Аннотация. Работа посвящена компьютерному моделированию процесса ползучести наследственных материалов. Процесс ползучести описывается нелинейным интегральным уравнением Ю. Н. Работнова, а ядро ползучести представлено ядром Абеля. Предложена новая эффективная методика определения параметров (α, δ) ядра Абеля. Для нахождения параметра α используется метод бисекции. Разработаны алгоритм и соответствующая компьютерная программа для вычисления параметров α и δ.

Разработаны эффективная методика и соответствующая компьютерная программа для вычисления значений условной мгновенной деформации и деформации ползучести наследственных материалов. На примерах материалов Nylon 6 и стеклопластика ТС 8/3-250 показана высокая точность моделирования процесса ползучести с помощью разработанных методик и компьютерных программ.

Ключевые слова: ползучесть, ядро Абеля, метод бисекции, условная мгновенная деформация, деформация ползучести.

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