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ҚАЗАҚСТАН РЕСПУБЛИКАСЫ  
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# Х А Б А Р Л А Р Ы

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## ИЗВЕСТИЯ

НАЦИОНАЛЬНОЙ АКАДЕМИИ НАУК  
РЕСПУБЛИКИ КАЗАХСТАН  
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## NEWS

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Satbayev University

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**BENDING OF AN ELASTIC THREE-LAYER PLATE  
WITH A HOLE CONNECTED TO THE SOIL FOUNDATION**

**Abstract.** The relevance of this paper is explained by a demand for the development of mechanical and mathematical models and methods for calculating the stress-strain state of the sandwich structural elements. The statement of the boundary value problem on the deformation of a circular sandwich plate with a central hole, connected to the soil foundation, was given. To describe the kinematics of an asymmetric plate pack, the broken line hypotheses are accepted. In a relatively thick lightweight core, the normal does not change its length, remains rectilinear, but rotates through some additional angle. Tuff, coarse grained soil, granite, and gneiss are accepted as the soil foundation. The bearing reaction is described by the Winkler model. The system of equilibrium equations is obtained by the variational method. Its solution is written in displacements through Kelvin functions. A numerical parametric analysis of displacements and stresses in the plate is carried out, their dependence on the type of soil foundation is shown.

**Key words:** three-layer plate with a hole, displacements, stresses, soil foundation.

**Introduction.** Currently, rods, plates and shells with a layered structure are usually assembled from materials with significantly different physical and mechanical properties. Bearing layers made of materials of high strength and rigidity (steel, duralumin, titanium, structural ceramics) are designed to absorb the main part of the mechanical loading. Bonding layer – core (foams, corrugated cardboard, polymer materials) serves to form a monolithic structure. This combination of layers makes it possible to ensure reliable operation of systems in adverse environmental conditions, to create structures that combine high strength and rigidity with a relatively low weight. For them, mathematical models of deformation under complex thermo-force, thermo-radiation loads are developed.

Numerous studies, including [1-13], have been devoted to the dynamics and vibrations of sandwich plates, including three-layer structural elements. Vibrations of sandwich cylindrical shells were considered in papers [1-5]. The solution to the problem of harmonic vibrations of a viscoelastoplastic sandwich cylindrical shell [1] was obtained using the expansion in a Fourier series and limiting only to its first term (monoharmonic approximation). Free and forced vibrations of sandwich cylindrical shells with elastic core were investigated in papers [2-4]. The solutions were written in the form of an expansion in double trigonometric series, the distribution of frequencies of free vibrations was analysed. The nonstationary problem of the plane oblique pressure wave diffraction on thin shell in the shape of parabolic cylinder was solved in [5]. Nonstationary contact problems associated with spherical shells were considered in [6-7].

Free and forced vibrations of sandwich rods and circular plates associated with an elastic foundation were considered in [8-13]. The kinematics of the elements correspond to the polyline hypothesis – in the bearing layers, Kirchhoff-Love hypotheses are valid, in a light core, the normal remains rectilinear, but rotates through some additional angle. The bearing reaction is adopted in the framework of the Winkler model. The solutions are obtained in the form of a series expansion in systems of orthonormal eigenfunctions. A numerical analysis of the dependence of the frequencies of free vibrations on the mechanical properties of materials and geometric parameters of structural elements is carried out. Thermal

and radiation impacts on a sandwich circular plate were considered. The papers [14-15] are devoted to the study of the strength of polymer composite panels with internal defects under the action of an unsteady loading. In the present paper the authors consider the bending of an elastic circular three-layer plate with a central hole connected to the Winkler base.

**Materials and methods.** The problem statement and its solution are carried out in a cylindrical coordinate system  $r, \varphi, z$ ;  $h_k$  denotes the relative thickness of the  $k$ -th layer. For isotropic bearing layers with thickness  $h_1, h_2$ , Kirchhoff-Love hypotheses are accepted. A core incompressible in thickness ( $h_3 = 2c$ ) is light, i.e., it neglects the work of shear stresses  $\sigma_{rz}$ . External vertical loading  $q_0=const$  is evenly distributed over the surface of the first layer. Movements are continuous at the layer boundaries. On the outer and inner contours of the plate (with radii  $r_0$  and  $r_1$ ), it is assumed that there is a rigid diaphragm that prevents the relative shift of the layers (figure 1).

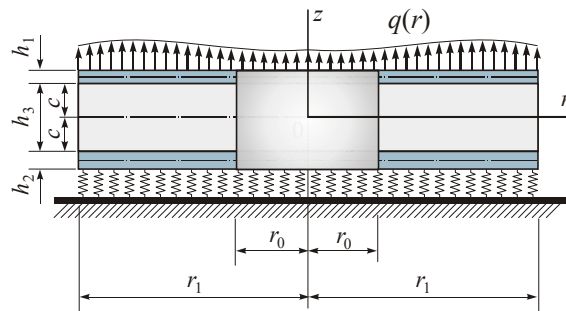


Figure 1 – Loading diagram

Using the hypothesis of the straightness of core normal:

$$2\varepsilon_{rz}^{(3)} = u_r^{(3)}, \tag{1}$$

$$z + w_{,r} = \psi, \tag{2}$$

after integration, obtain equations for the radial displacements in the layers  $u_r^{(k)}$  in terms of the required functions:

$$u_r^{(2)} = u - c\psi - zw_{,r} (-c - h_2 \leq z \leq -c), \tag{3}$$

where  $z$  – the coordinate of the fibre under consideration, the subscript comma denotes the operation of differentiation by the next coordinate.

Deformations in the layers follow from equation (3) and Cauchy relations [16] in a cylindrical coordinate system. It is assumed that the relationship between stresses and strains in the layers is described by the relations of the linear theory of elasticity in the deviatoric and spherical form:

$$s_\alpha^k = 2G_k \varepsilon_\alpha^k, \tag{6}$$

$$\sigma^k = 3K_k \varepsilon^k (\alpha = r, \varphi), \tag{7}$$

$$s_{sc}^3 = 2G_3 e_{sc}^3 = G_3 \psi, \tag{8}$$

where  $s_\alpha^k, e_\alpha^k$  and  $\sigma^k, \varepsilon^k$  – deviatoric and spherical parts of stress and strain tensors,  $s_{sc}^3 = \sigma_{sc}^3$  – shear stress in the core;  $G_k, K_k$  – shear and volumetric deformation moduli in the  $k$ -th layer. Using the stress tensor components

$$\sigma_\alpha^k = S_\alpha^k + \sigma^k (\alpha = r, \varphi), \tag{9}$$

introduce generalised internal forces and moments in the plate:

$$T_\alpha \equiv \sum_{k=1}^3 T_\alpha^{(k)} = \sum_{k=1}^3 \int_{h_k} \sigma_\alpha^{(k)} dz, \tag{10}$$

$$M_{\alpha} \equiv \sum_{k=1}^3 M_{\alpha}^{(k)} = \sum_{k=1}^3 \int_{h_k} \sigma_{\alpha}^{(k)} z dz, \quad (11)$$

$$H_{\alpha} = M_{\alpha}^{(3)} + c(T_{\alpha}^{(1)} - T_{\alpha}^{(2)}). \quad (12)$$

A system of differential equations in the forces describing the equilibrium of the considered circular sandwich plate on an elastic foundation is obtained:

$$T_{r,r} + \frac{1}{r}(T_r - T_{\varphi}) = -p, \quad (13)$$

$$H_{r,r} + \frac{1}{r}(H_r - H_{\varphi}) = 0, \quad (14)$$

$$M_{r,rr} + \frac{1}{r}(2M_{r,r} - M_{\varphi,r}) = -q_0 + q_R. \quad (15)$$

On the contours of the plate ( $r = r_0$  and  $r = 1$ ), the force boundary conditions must be satisfied

$$T_r = T_r^n, \quad H_r = H_r^n, \quad M_r = M_r^n, \quad M_{r,r} + \frac{1}{r}(M_r - M_{\varphi}) = Q^n. \quad (16)$$

Obtain an equation for the generalised efforts included in (13) – (15) in terms of the three required functions  $u(r)$ ,  $\psi(r)$ ,  $w(r)$ . For this, substitute into (10) – (12) the equation of stresses through deformations (6) – (8), and then deformations through displacement.

The relations for  $T_{\varphi}$ ,  $M_{\varphi}$ ,  $H_{\varphi}$  follow from the formulas for  $T_r$ ,  $M_r$ ,  $H_r$ , if in (8) to swap  $K_k^+$  and  $K_k^-$ . After substitution of equation in (13) – (16), obtain a system of differential equations in displacements describing the bending of an annular three-layer plate with a light core on an elastic foundation:

$$L_2(a_1 u + a_2 \psi - a_3 w_{,r}) = 0, \quad (17)$$

$$L_2(a_2 u + a_4 \psi - a_5 w_{,r}) = 0, \quad (18)$$

$$L_3(a_3 u + a_5 \psi - a_6 w_{,r}) - \kappa_0 w = -q_0, \quad (19)$$

where  $q_0$  – external distributed load intensity.

The problem of finding the functions  $u(r)$ ,  $\psi(r)$ ,  $w(r)$  is closed by adding boundary conditions to equations (17) – (19). For rigid constraint of the contours of the plate ( $r = r_0, 1$ )

$$u = \psi = w = w_{,r} = 0. \quad (20)$$

With hinged contours ( $r = r_0, 1$ )

$$u = \psi = w = M_r = 0. \quad (21)$$

The system of equations (17) – (19) after transformations is reduced to the form

$$u = b_1 w_{,r} + C_1 r + C_2 / r, \quad (22)$$

$$\psi = b_2 w_{,r} + C_3 r + C_4 / r, \quad (23)$$

$$w_{,rrrr} + \frac{2}{r} w_{,rrr} - \frac{1}{r^2} w_{,rr} + \frac{1}{r^3} w_{,r} + \kappa^4 w = q, \quad (24)$$

where  $C_1, C_2, C_3, C_4$  – integration constants.

$$D = \frac{a_1(a_1 a_4 - a_2^2)}{(a_1 a_6 - a_3^2)(a_1 a_4 - a_2^2) - (a_1 a_5 - a_2 a_3)^2}, \quad (25)$$



$$\kappa^4 = \kappa_0 D, \quad (26)$$

$$q = q_0 D, \quad (27)$$

$$b_1 = \frac{a_3 a_4 - a_2 a_5}{a_1 a_4 - a_2^2}, \quad (28)$$

$$b_2 = \frac{a_1 a_5 - a_2 a_3}{a_1 a_4 - a_2^2}. \quad (29)$$

The general solution of the third equation in (31) will be

$$w = C_5 \operatorname{ber}(\kappa r) + C_6 \operatorname{bei}(\kappa r) + C_7 \operatorname{ker}(\kappa r) + C_8 \operatorname{kei}(\kappa r) + w_0, \quad (30)$$

where the zero-order Kelvin functions  $\varphi_n(\kappa r) = \operatorname{ber}(\kappa r), \operatorname{bei}(\kappa r), \operatorname{ker}(\kappa r), \operatorname{kei}(\kappa r)$  form a fundamental system of solutions,  $w_0$  is a particular solution of the inhomogeneous equation (27), depending on the form of the right-hand side, i.e., the loading. As a result, from (27), taking into account (28), obtain the desired displacements in an elastic sandwich circular plate on a deformable base:

$$u = b_1 w_{,r} + C_1 r + C_2 / r, \quad (31)$$

$$\psi = b_2 w_{,r} + C_3 r + C_4 / r, \quad (32)$$

$$w = C_5 \operatorname{ber}(\kappa r) + C_6 \operatorname{bei}(\kappa r) + C_7 \operatorname{ker}(\kappa r) + C_8 \operatorname{kei}(\kappa r) + \frac{q_0}{\kappa_0}. \quad (33)$$

Integration constants  $C_1, C_2, \dots, C_8$ , corresponding in the most general case to eight boundary conditions, are determined in each particular case of fixing the outer and inner contours of the plate. In the future, expressions for the first two derivatives of the deflection and their values on the boundary contours will be necessary. In accordance with the rule of differentiation of Kelvin functions [6], the derivative of the deflection in (29) – (31) will be as follows:

$$w_{,r} = \frac{\kappa \sqrt{2}}{2} \{C_5 [\operatorname{ber}_1(\kappa r) + \operatorname{bei}_1(\kappa r)] + C_6 [-\operatorname{ber}_1(\kappa r) + \operatorname{bei}_1(\kappa r)] + C_7 [\operatorname{ker}_1(\kappa r) + \operatorname{kei}_1(\kappa r)] + C_8 [-\operatorname{ker}_1(\kappa r) + \operatorname{kei}_1(\kappa r)]\}, \quad (34)$$

Differentiating the first derivative (36) with respect to the radius, obtain the second derivative:

$$w_{,rr} = \frac{\kappa^2}{2} \{C_5 [\operatorname{bei}_2(\kappa r) - \operatorname{ber}_2(\kappa r)] + C_6 [-\operatorname{ber}_2(\kappa r) + \operatorname{ber}(\kappa r)] + C_7 [\operatorname{kei}_2(\kappa r) - \operatorname{kei}(\kappa r)] + C_8 [-\operatorname{ker}_2(\kappa r) + \operatorname{ker}(\kappa r)]\}, \quad (35)$$

The value of the second derivative (37) on the contours ( $r = 1, r = r_0$ ):

$$w_{,rr}(1) = b_5 C_5 + b_6 C_6 + b_{50} C_7 + b_{60} C_8, \quad (36)$$

$$w_{,rr}(r_0) = b_{51} C_5 + b_{61} C_6 + b_{52} C_7 + b_{62} C_8, \quad (37)$$

In the case of *rigid constraint of both plate contours*, equation (34) must be substituted in (21). As a result, taking into account (57) and the fact that the values of the derivative of the deflection on the contours of the plate are equal to zero, obtain a linear system of eight algebraic equations. From the first four it follows that  $C_1 = C_2 = C_3 = C_4 = 0$ . From the remaining equations

$$C_5 \operatorname{ber} \kappa + C_6 \operatorname{bei} \kappa + C_7 \operatorname{ker} \kappa + C_8 \operatorname{kei} \kappa = -q_0 / \kappa_0, \quad (38)$$

$$C_5 \operatorname{ber}(\kappa r_0) + C_6 \operatorname{bei}(\kappa r_0) + C_7 \operatorname{ker}(\kappa r_0) + C_8 \operatorname{kei}(\kappa r_0) = -q_0 / \kappa_0, \quad (39)$$

$$b_3C_5 + b_4C_6 + b_{30}C_7 + b_{40}C_8 = 0, \quad (40)$$

$$b_{31}C_5 + b_{41}C_6 + b_{32}C_7 + b_{42}C_8 = 0. \quad (41)$$

the integration constants can be determined either using determinants or numerically.

If both contours of the plate are hinged, then (34) must be substituted in (22). As a result, the following algebraic system of equations for determining the integration constants is obtained:

$$C_1 + C_2 + b_1(b_3C_5 + b_4C_6 + b_{30}C_7 + b_{40}C_8) = 0, \quad (42)$$

$$C_1r_0 + C_2/r_0 + b_1(b_{31}C_5 + b_{41}C_6 + b_{32}C_7 + b_{42}C_8) = 0, \quad (43)$$

$$C_3 + C_4 + b_2(b_3C_5 + b_4C_6 + b_{30}C_7 + b_{40}C_8) = 0, \quad (44)$$

$$C_3r_0 + C_4/r_0 + b_2(b_{31}C_5 + b_{41}C_6 + b_{32}C_7 + b_{42}C_8) = 0, \quad (45)$$

$$C_5 \operatorname{ber} \kappa + C_6 \operatorname{bei} \kappa + C_7 \operatorname{ker} \kappa + C_8 \operatorname{kei} \kappa = -q_0 / \kappa_0, \quad (46)$$

$$C_5 \operatorname{ber}(\kappa r_0) + C_6 \operatorname{bei}(\kappa r_0) + C_7 \operatorname{ker}(\kappa r_0) + C_8 \operatorname{kei}(\kappa r_0) = -q_0 / \kappa_0, \quad (47)$$

$$a_3C_1 - a_3C_2 + a_5C_3 - a_5C_4 + b_{71}C_5 + b_{72}C_6 + b_{73}C_7 + b_{74}C_8 = 0, \quad (48)$$

$$a_3C_1 - a_3C_2 + a_5C_3 - a_5C_4 + b_{81}C_5 + b_{82}C_6 + b_{83}C_7 + b_{84}C_8 = 0, \quad (49)$$

The solution of the system of equations (44) – (49) gives the desired integration constants. Thus, equation (36) describes displacements in a circular sandwich plate with a hole connected to an elastic base in the case of constraint or hinged support of both contours.

**Results and discussion.** Figure 2, *a, b* shows the change along the radius of the plate deflection  $w$  and shear in the core  $\psi$ . The intensity of the surface loading on the plate to the base was taken as  $q_0 = 10$  MPa. The curves are plotted at different coefficients of soil reaction (MPa/m): 1 –  $\kappa_0 = 100$ , 2 –  $\kappa_0 = 1000$ , 3 –  $\kappa_0 = 10000$ . An increase in the stiffness coefficient of the soil leads to a decrease in displacements in magnitude, but the shape of the curves remains, as opposed to a solid plate. This is due to the high rigidity of the circular plate itself, both boundary contours of which are clamped [17-21].

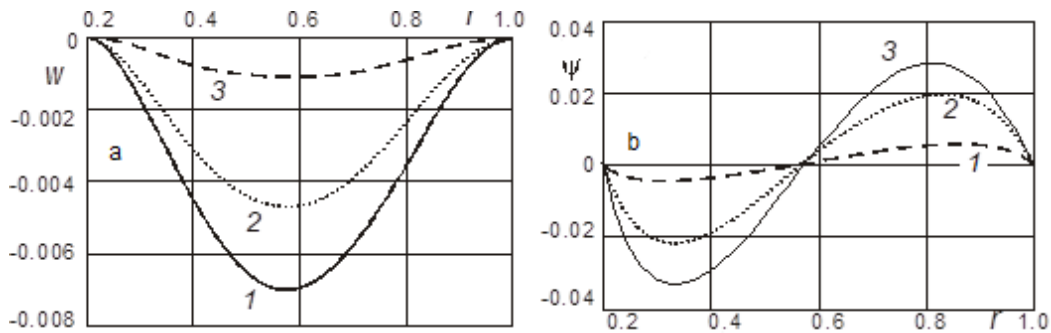


Figure 2 – Change in deflection  $w$  and shear in the core  $\psi$  along the plate radius

Figure 3 shows the change in radial  $\sigma_r - 1$  and circumferential  $\sigma_\phi - 2$  stresses along the thickness of the plate on its outer – *a* and inner – *b* contours. The stiffness coefficient of the base is  $\kappa = 1000$  MPa/m. On the outer contour, the upper parts of the bearing layers are compressed, the core is stretched (due to the edge effect). On the inner contour, the opposite is true. In the gluing of the layers, the stresses have the same signs, but they break due to the different mechanical characteristics of the materials [22-27]. The stress pattern is symmetrical due to the symmetry of plate thickness. In both cases, the highest stresses are achieved on the outer planes of the plate, while they are equal in magnitude. In the bearing layers, the stress values are reduced by 109, in the core – by 108 times. The intensity of the external load is taken as  $q = -10$  MPa, the radius of the inner boundary contour  $r_0 = 0.2$ .

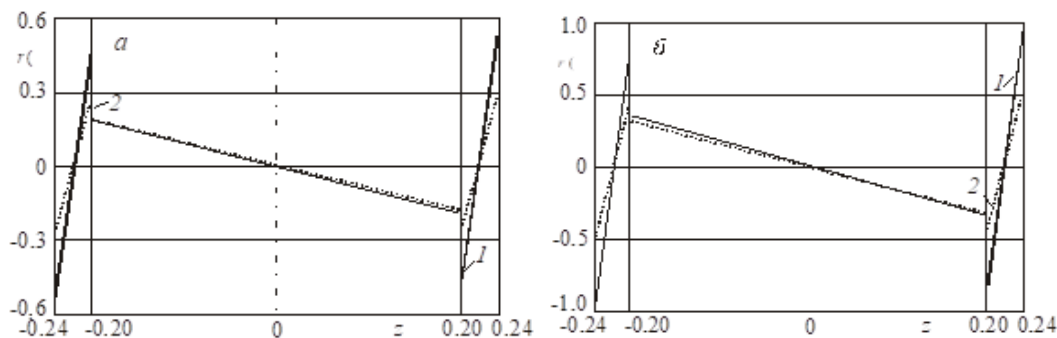


Figure 3 – Change of radial – 1 and circumferential – 2 stresses along the plate thickness

**Conclusions.** Numerical studies are carried out for a plate, the layers of which are made of D16T–fluoroplastic-4–D16T materials. Dust sand, coarse-grained soil, and granite were assumed as the base. They showed that with an increase in the stiffness of the foundation, the maximum deflection and relative shear decrease significantly. The stresses reach their maximum on the inner contour, where they are 1.5 times the stresses on the outer contour. With an increase in the stiffness of the base to a high, the maximum stresses decrease in modulus by a factor of 2.6.

Thus, the proposed mathematical model makes it possible to study the stress-strain state of elastic sandwich circular plates with a central hole connected to an elastic soil foundation of arbitrary stiffness, with different methods of fixing its external and internal contours of any axisymmetric loads. The obtained analytical solution can be used to carry out the corresponding numerical experiments when calculating composite structural elements in construction and mechanical engineering.

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#### ТОПЫРАҚТЫ НЕГІЗБЕН БАЙЛАНЫСҚАН САҢЫЛАУЫ БАР СЕРПІМДІ ҮШҚАБАТТЫ ТІЛІМ ІЛІСІ

**Аннотация.** Жұмыстың өзектілігі сэндвич құрылым бөлшектерінің кернеулі-деформацияланған күйінің механикалық және математикалық модельдері мен есептеу әдісін жасаудың сұраныста болғандығымен түсіндіріледі. Топырақты негізбен қосылған, ортасында саңылауы бар дөңгелек көпқабатты тақтаның деформациялануына қатысты негізгі міндеті айқындалды. Асимметриялық тілімнің десте кинематикасын сипаттау үшін сынық сызық гипотезасы есепке алынды. Салыстырмалы жуан және жеңіл өзекте норма ұзындығы өзгермейді, түзу болып қалады және кейбір қосымша бұрыштарға қайырылады. Топырақ негізі – туф, ірітүйіршікті топырақ, гранит пен гнейс. Мойынтірек әсері Винклер моделімен сипатталады. Тепе-теңдік теңдеуінің жүйесі вариациялық әдіс арқылы алынды. Оның шешімі жылжу кезіндегі Кельвин функциясымен жазылып отырады. Топырақты негіздің түріне тәуелділігін көрсететін тақтадағы жылжу мен кернеудің сандық параметрлік талдамасы жасалды.

**Түйін сөздер:** саңылауы бар үшқабатты тілім, жылжу, кернеу, топырақты негіз.

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### **ИЗГИБ УПРУГОЙ ТРЕХСЛОЙНОЙ ПЛАСТИНЫ С ОТВЕРСТИЕМ, СВЯЗАННОЙ С ГРУНТОВЫМ ОСНОВАНИЕМ**

**Аннотация.** Актуальность данной работы объясняется востребованностью разработки механических и математических моделей и методов расчета напряженно-деформированного состояния элементов конструкции сэндвича. Дана постановка краевой задачи о деформировании круглой многослойной плиты с центральным отверстием, соединенным с грунтовым основанием. Для описания кинематики пакета асимметричных пластин приняты гипотезы ломаной линии. В относительно толстом легком ядре норма не меняет длину, остается прямой, а поворачивается на некоторые дополнительные углы. Почвенная основа – туф, крупнозернистый грунт, гранит и гнейс. Реакция подшипника описывается моделью Винклера. Система уравнений равновесия получена вариационным методом. Ее решение записывается функциями Кельвина при перемещении. Проведен количественный параметрический анализ перемещений и напряжений в плите, показывающий их зависимость от типа грунтового основания.

**Ключевые слова:** трехслойная пластина с отверстиями, перемещения, напряжения, грунтовое основание.

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